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The order parameters of a spin-1 Ising film in a transverse field

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Abstract. Using the effective-field theory with a probability distribution technique that accounts for the self-spin-correlation functions, the layer longitudinal magnetizations and quadrupolar moments of a spin-1 Ising film and their averages are examined. These quantities as functions of the temperature, the ratio of the surface exchange interactions to the bulk ones, the strength of the transverse field and the film thickness are calculated numerically and some interesting results are obtained.

1. Introduction

The development of the molecular-beam-epitaxy technique and its application to the growth of thin metallic films has simulated renewed interest in both experimental and theoretical thin-film magnetism [1, 2]. Magnetic films can be studied as models of the magnetic size effect and quasi-two-dimensional systems [3]. Part of this theoretical activity has been devoted to the study of the magnetic and phase transition properties of semi-infinite Ising systems and magnetic films [4–25]. The surface magnetism of these systems is very interesting [3, 4]. When the ratio of the surface exchange interactions to the bulk ones $R = J_s/J$ is larger than a critical value R_c, the system becomes ordered on the surface before it is ordered in the bulk, and the critical temperature of the surface T_c^S/J is higher than that of the bulk T_c^B/J. For a film composed of a number L of atomic layers parallel to the surfaces, one cannot differentiate between the critical temperatures of the surface and the internal layers, and the film has a unified critical temperature T_c/J [6–11, 25–26]. For a film in which there are a great number of atomic layers, the magnetism is similar to that of a semi-infinite system [6–11, 25], in the sense that there exists a critical value R_c of the parameter R such that when $R \leqslant R_c$ the film critical temperature is smaller than that of the bulk system and approaches the latter for large values of the film thickness L and when $R \geqslant R_c$ the film critical temperature is larger both than the surface and than the bulk critical temperatures of the corresponding semi-infinite system and approaches the surface critical temperature for large values of the film thickness. The critical properties of ordinary Ising films, transverse Ising films and diluted transverse Ising films have been calculated and discussed [1, 2, 5–26]. In some cases, magnetic properties such as the magnetizations and the susceptibilities have also been investigated. Apart from references [4, 7], all of the studies mentioned above are concerned with Ising spin systems of magnitude $\frac{1}{2}$.

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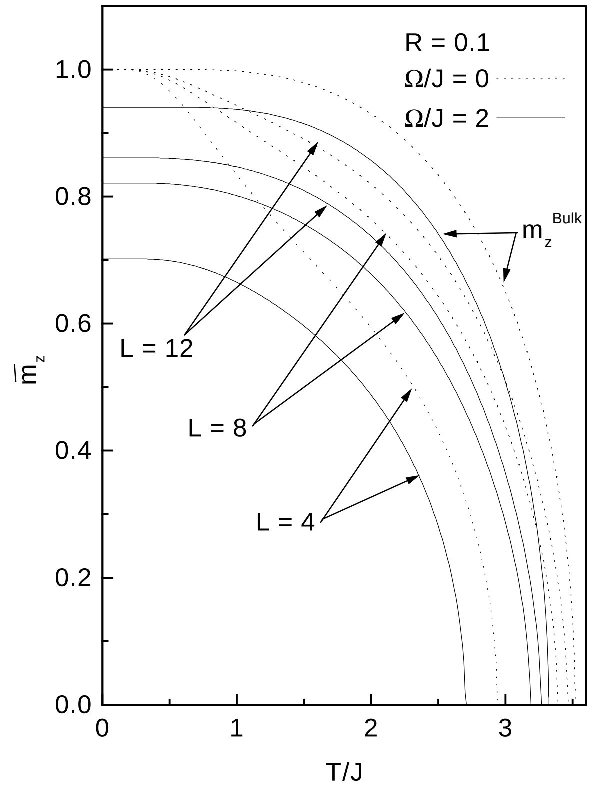
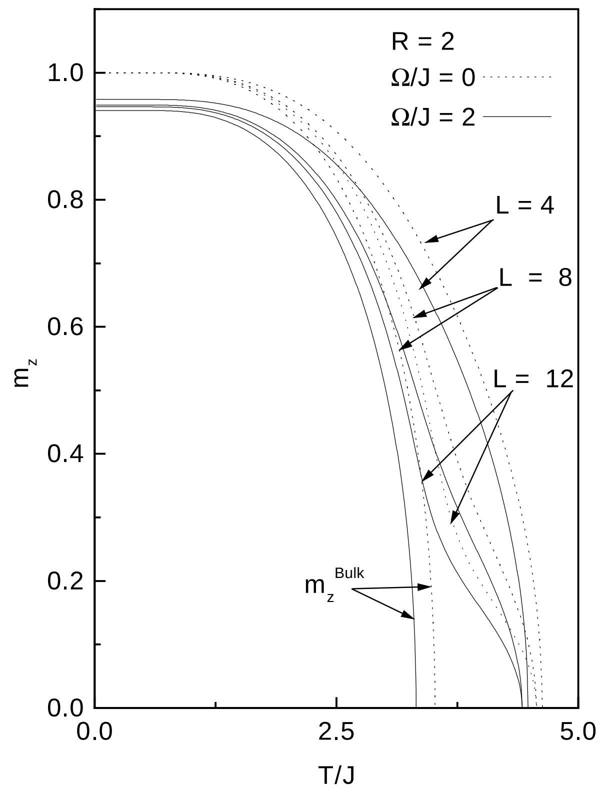


(a)

Figure 1. The film longitudinal magnetization as a function of the temperature. (a) $R = 0.1$, (b) $R = 2$. The dotted and solid curves correspond respectively to $\Omega/J = 0$ and 2.

In addition, there have not been many studies of the critical properties and magnetic properties of the transverse Ising film with a higher spin. To our knowledge, Benyoussef *et al* [4] have studied the semi-infinite spin-1 Ising model using the mean-field approximation and Zhong *et al* [7], within the framework of the effective-field theory with the differential operator technique, have investigated the critical behaviour of a transverse Ising ferromagnetic thin film with spin 1, but unfortunately in their calculations the derivation of the expression for the longitudinal quadrupolar moment is incorrect and consequently this yields incorrect phase diagrams.

Our aim in this paper is to study the layer longitudinal magnetizations and quadrupolar moments of a spin-1 Ising film in a uniform transverse field within the framework of the effective-field theory using a probability distribution technique [27, 28]. This technique is believed to give more exact results than the standard mean-field approximation. In section 2, we outline the formalism and derive the equations that determine the layer magnetizations and quadrupolar moments, and their averages. The longitudinal magnetizations and quadrupolar moment curves of the film as functions of the temperature, the ratio of the surface exchange interactions to the bulk ones, the strength of the transverse field and the film thickness are discussed in section 3 and comparison has been made with the results for the longitudinal magnetizations obtained by Wang *et al* [11] and Ainane *et al* [26] for the spin-$\frac{1}{2}$ case. The last section, section 4, is devoted to a brief conclusion.



(b)

Figure 1. (Continued)

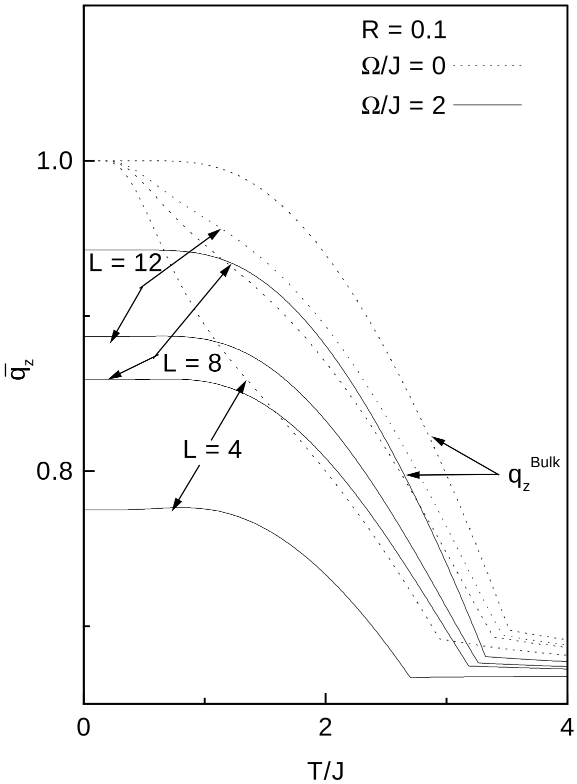
2. Formalism

We consider a spin-1 Ising film of L layers on a simple cubic lattice with free surfaces parallel to the (001) plane, in a uniform transverse field Ω. The Hamiltonian of the system is given by

$$H = -\sum_{(i,j)} J_{ij} S_{iz} S_{jz} - \Omega \sum_{i} S_{ix} \tag{1}$$

where S_{iz} and S_{ix} denote the z- and x-components of a quantum spin $\vec{S}_i$ of magnitude $S = 1$ at site i, and $J_{i,j}$ is the strength of the exchange interaction between the spins at nearest-neighbour sites i and j, which is equal to J_s if both spins i and j are on the surface layers and to J otherwise. The statistical properties of the system are studied using an effective-field theory that employs the probability distribution technique, which accounts for the single-site correlations. This technique has been applied successfully to the study of various physical problems and in particular to the transverse spin-S Ising system of magnitude S [27]. In the case of the spin-1 Ising model in a transverse field, following reference [27], we find in the current situation for a fixed configuration of neighbouring spins of the site i that the longitudinal magnetization and quadrupolar moment of any spin at site i are given, where

$$\mathcal{S} = \sum_{j} J_{ij} S_{jz}$$



(a)

Figure 2. The film longitudinal quadrupolar moment as a function of the temperature. (a) $R = 0.1$, (b) $R = 2$. The dotted and solid curves correspond respectively to $\Omega/J = 0$ and 2.

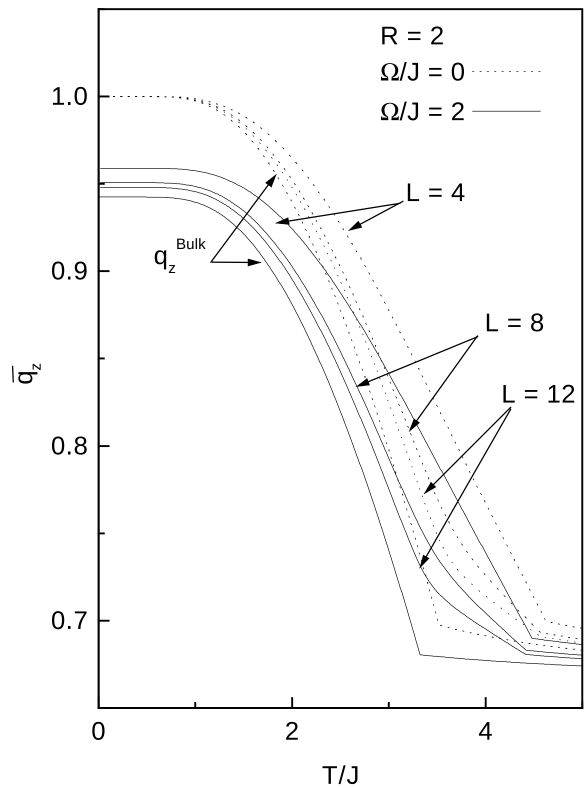
by

$$m_{iz} = \langle S_{iz}\rangle = \left\langle \frac{\mathcal{S}}{[\mathcal{S}^2 + \Omega^2]^{1/2}} \frac{2\sinh(\beta[\mathcal{S}^2 + \Omega^2]^{1/2})}{1 + 2\cosh(\beta[\mathcal{S}^2 + \Omega^2]^{1/2})} \right\rangle = \langle f_{1z}[\mathcal{S}, \Omega]\rangle \tag{2}$$

$$q_{iz} = \langle (S_{iz})^2\rangle = \left\langle \frac{1}{\mathcal{S}^2 + \Omega^2} \frac{\Omega^2 + [2\mathcal{S}^2 + \Omega^2]\cosh(\beta[\mathcal{S}^2 + \Omega^2]^{1/2})}{1 + 2\cosh(\beta[\mathcal{S}^2 + \Omega^2]^{1/2})} \right\rangle = \langle f_{2z}[\mathcal{S}, \Omega]\rangle \tag{3}$$

where m_{iz} and q_{iz} are respectively the longitudinal magnetization and quadrupolar order parameter of the ith site, $\beta = 1/k_B T$, $\langle\cdots\rangle$ indicates the usual canonical ensemble thermal average for a given configuration and the sum is over all of the nearest neighbours of the site i. The expression for q_{iz} given by equation (3) is different from the corresponding expression derived by Zhong *et al* [7], where the expression is incorrect. The correct derivation of equation (3) has been presented by Elkouraychi *et al* [27].

In a mean-field approximation, one would simply replace these spin operators by their thermal values m_z (the longitudinal magnetizations). However, it is at this point that a substantial improvement to the theory is made by noting that the spin operators have a finite set of base states, with the result that the averages over the functions f_{1z} and f_{2z} can be expressed as an average over a finite polynomial of spin operators belonging to the neighbouring spins. This procedure can be effected by the combinatorial method and correctly accounts for the single-site kinematic relations. Up to this point, the right-hand sides of equations (2) and (3)



(b)

Figure 2. (Continued)

will contain multiple spin-correlation functions. To perform thermal averaging on the right-hand side of equations (2) and (3), one now follows references [27, 28]. Thus, with the use of the integral representation method of the Dirac δ-distribution, equations (2) and (3) can be written in the form

$$m_{iz} = \int \mathrm{d}\omega \; f_{1z}(\omega, \Omega) \frac{1}{2\pi} \int \mathrm{d}t \;\; \exp(\mathrm{i}\omega t) \prod_j \langle \exp(-\mathrm{i}t J_{ij} S_{jz}) \rangle \tag{4}$$

$$q_{iz} = \int \mathrm{d}\omega \; f_{2z}(\omega, \Omega) \frac{1}{2\pi} \int \mathrm{d}t \exp(\mathrm{i}\omega t) \prod_j \langle \exp(-\mathrm{i}t J_{ij} S_{jz}) \rangle \tag{5}$$

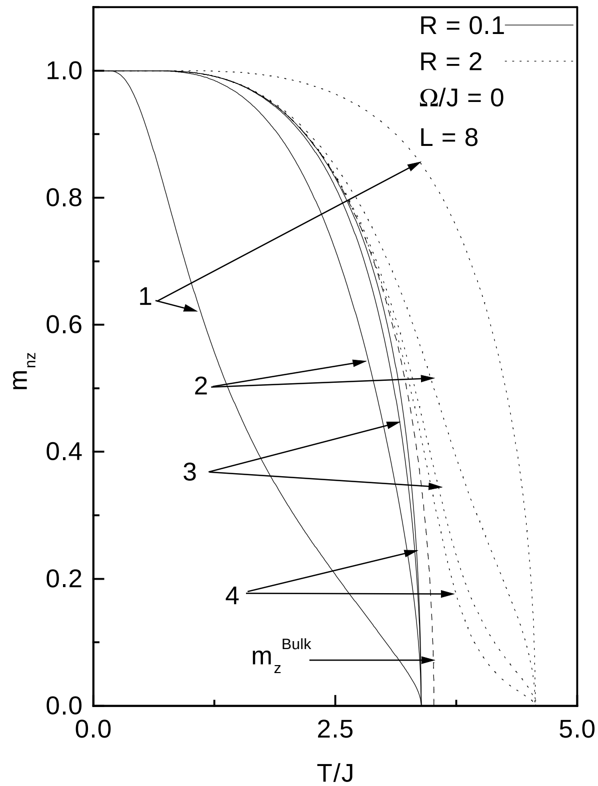
where

$$f_{1z}(y, \Omega) = \frac{y}{[y^2 + \Omega^2]^{1/2}} \frac{2 \sinh(\beta [y^2 + \Omega^2]^{1/2})}{1 + 2 \cosh(\beta [y^2 + \Omega^2]^{1/2})} \tag{6}$$

$$f_{2z}(y, \Omega) = \frac{1}{[y^2 + \Omega^2]} \frac{\Omega^2 + (2y^2 + \Omega^2) \cosh(\beta [y^2 + \Omega^2]^{1/2})}{1 + 2 \cosh(\beta [y^2 + \Omega^2]^{1/2})}. \tag{7}$$

In the derivation of equations (4) and (5), the commonly used approximation has been made according to which the multi-spin-correlation functions are decoupled into products of the spin averages (the simplest approximation of neglecting the correlations between different sites has been made). That is

$$\langle S_{j_1} S_{j_2} S_{j_3}^2 S_{j_4}^2 S_{j_5} \cdots \rangle = \langle S_{j_1} \rangle \langle S_{j_2} \rangle \langle S_{j_3}^2 \rangle \langle S_{j_4}^2 \rangle \langle S_{j_5} \rangle \cdots . \tag{8}$$



(a)

Figure 3. Layer longitudinal magnetizations as functions of the temperature for a film with $L = 8$ layers. (a) $\Omega/J = 0$, (b) $\Omega/J = 2$. The solid and dotted curves correspond respectively to $R = 0.1$ and 2.

To make progress, we introduce the probability distribution of the spin variables S_{iz}, which is given in [27] by

$$P(S_{iz}) = \frac{1}{2}[(q_{iz} - m_{iz})\delta(S_{iz} + 1) + 2(1 - q_{iz})\delta(S_{iz}) + (q_{iz} + m_{iz})\delta(S_{iz} - 1)] \tag{9}$$

with

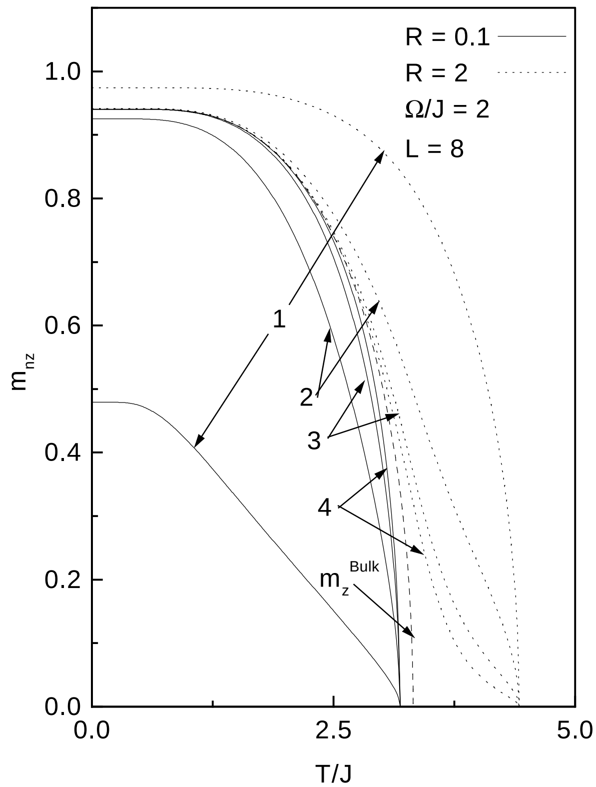
$$m_{iz} = \langle S_{iz} \rangle \tag{10}$$

and

$$q_{iz} = \langle S_{iz}^2 \rangle. \tag{11}$$

Allowing the site magnetizations and quadrupolar moments to take different values in each atomic layer parallel to the surface of the film, and labelling them in accordance with the layer in which they are situated, the application of equations (2), (4) and (9) yields the following set of equations for the layer longitudinal magnetizations:

$$\begin{aligned} m_{1z} = 2^{-N-N_0} \sum_{\mu=0}^{N} \sum_{\nu=0}^{N-\mu} \sum_{\mu_1=0}^{N_0} \sum_{\nu_1=0}^{N_0-\mu_1} & 2^{\mu+\mu_1} C_{\mu}^{N} C_{\nu}^{N-\mu} C_{\mu_1}^{N_0} C_{\nu_1}^{N_0-\mu_1} (1 - 2q_{1z})^{\mu} (q_{1z} - m_{1z})^{\nu} \\ & \times (q_{1z} + m_{1z})^{N-\mu-\nu} (1 - 2q_{2z})^{\mu_1} (q_{2z} - m_{2z})^{\nu_1} (q_{2z} + m_{2z})^{N_0-\mu_1-\nu_1} \\ & \times f_{1z}(J[R(N - \mu - 2\nu) + (N_0 - \mu_1 - 2\nu_1)], \Omega) \end{aligned} \tag{12}$$



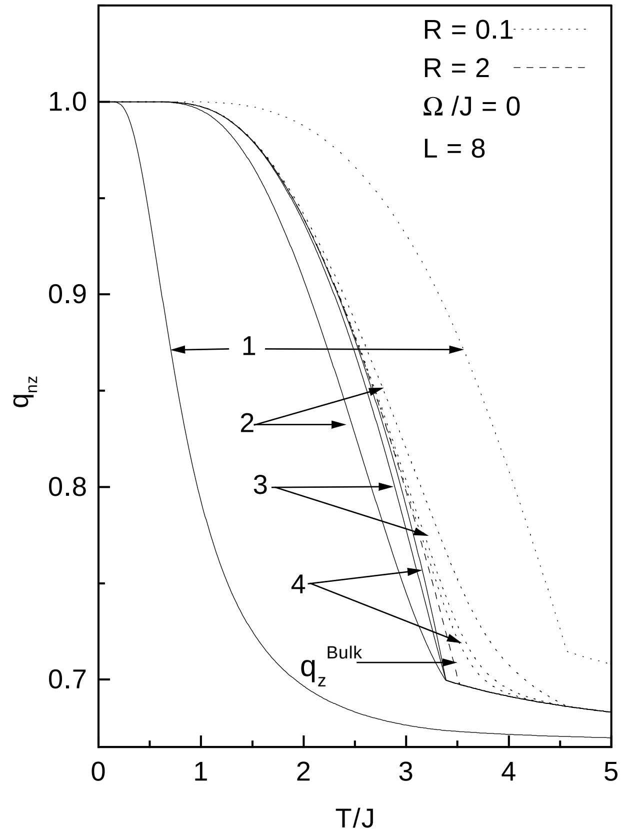
(b)

Figure 3. (Continued)

$$m_{nz} = 2^{-N-2N_0} \sum_{\mu=0}^{N} \sum_{\nu=0}^{N-\mu} \sum_{\mu_1=0}^{N_0} \sum_{\nu_1=0}^{N_0-\mu_1} \sum_{\mu_2=0}^{N_0} \sum_{\nu_2=0}^{N_0-\mu_2} 2^{\mu+\mu_1+\mu_2} C_\mu^N C_\nu^{N-\mu} C_{\mu_1}^{N_0} C_{\nu_1}^{N_0-\mu_1} C_{\mu_2}^{N_0} C_{\nu_2}^{N_0-\mu_2}$$
$$\times (1-2q_{nz})^\mu (q_{nz} - m_{nz})^\nu (q_{nz} + m_{nz})^{N-\mu-\nu} (1-2q_{n-1,z})^{\mu_1}$$
$$\times (q_{n-1,z} - m_{n-1,z})^{\nu_1} (q_{n-1,z} + m_{n-1,z})^{N_0-\mu_1-\nu_1} (1-2q_{n+1,z})^{\mu_2}$$
$$\times (q_{n+1,z} - m_{n+1,z})^{\nu_2} (q_{n+1,z} + m_{n+1,z})^{N_0-\mu_2-\nu_2}$$
$$\times f_{1z}(J[R(N-\mu-2\nu) + (2N_0 - \mu_1 - \mu_2 - 2\nu_1 - 2\nu_2)], \Omega)$$
$$\text{for } n = 2, 3, \ldots, L-1 \tag{13}$$

$$m_{Lz} = 2^{-N-N_0} \sum_{\mu=0}^{N} \sum_{\nu=0}^{N-\mu} \sum_{\mu_1=0}^{N_0} \sum_{\nu_1=0}^{N_0-\mu_1} 2^{\mu+\mu_1} C_\mu^N C_\nu^{N-\mu} C_{\mu_1}^{N_0} C_{\nu_1}^{N_0-\mu_1} (1-2q_{Lz})^\mu (q_{Lz} - m_{Lz})^\nu$$
$$\times (q_{Lz} + m_{Lz})^{N-\mu-\nu} (1-2q_{L-1,z})^{\mu_1} (q_{L-1,z} - m_{L-1,z})^{\nu_1}$$
$$\times (q_{L-1,z} + m_{L-1,z})^{N_0-\mu_1-\nu_1} f_{1z}(J[R(N-\mu-2\nu) + (N_0 - \mu_1 - 2\nu_1)], \Omega) \tag{14}$$

where N and N_0 are the numbers of nearest neighbours in the plane and between adjacent planes respectively ($N = 4$ and $N_0 = 1$ in the case of a simple cubic lattice which is considered here) and C_k^l are the binomial coefficients, $C_k^l = l!/(k!(l-k)!)$.



(a)

Figure 4. Layer longitudinal quadrupolar moments as functions of the temperature for a film with $L = 8$ layers. (a) $\Omega/J = 0$, (b) $\Omega/J = 2$. The solid and dotted curves correspond respectively to $R = 0.1$ and 2.

The equations of the longitudinal quadrupolar moments are obtained by substituting for the function f_{1z} with f_{2z} in the expressions of the layer longitudinal magnetizations. This yields

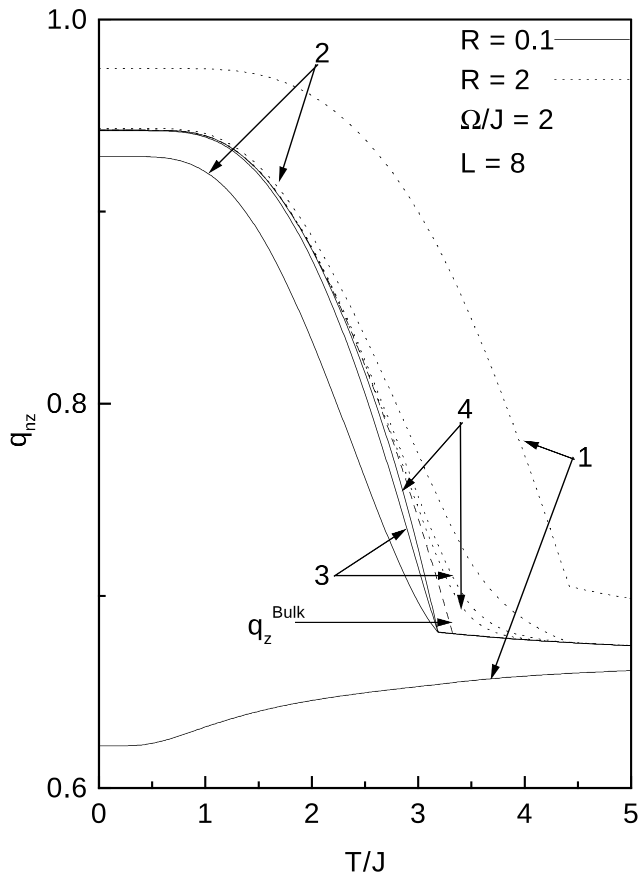
$$q_{nz} = m_{nz}[f_{1\alpha}(y, \Omega) \rightarrow f_{2z}(y, \Omega)]. \tag{15}$$

We have thus obtained the self-consistent equations (12)–(15) for the layer longitudinal magnetizations m_{nz} and quadrupolar moments q_{nz}, which can be solved directly by numerical iteration. No further algebraic manipulation is necessary.

The corresponding layer transverse magnetizations and quadrupolar moments, m_{nx} and q_{nx}, can be obtained from the longitudinal ones by interchanging $\sum_j J_{ij} S_{jz}$ and Ω. This yields

$$m_{nx} = m_{nz}\left\{f_{1z}\left[\left(\sum_j J_{ij} S_{jz}\right), \Omega\right] \rightarrow f_{1z}\left[\Omega, \left(\sum_j J_{ij} S_{jz}\right)\right]\right\} \tag{16}$$

$$q_{nx} = q_{nz}\left\{f_{2z}\left[\left(\sum_j J_{ij} S_{jz}\right), \Omega\right] \rightarrow f_{2z}\left[\Omega, \left(\sum_j J_{ij} S_{jz}\right)\right]\right\}. \tag{17}$$



(b)

Figure 4. (Continued)

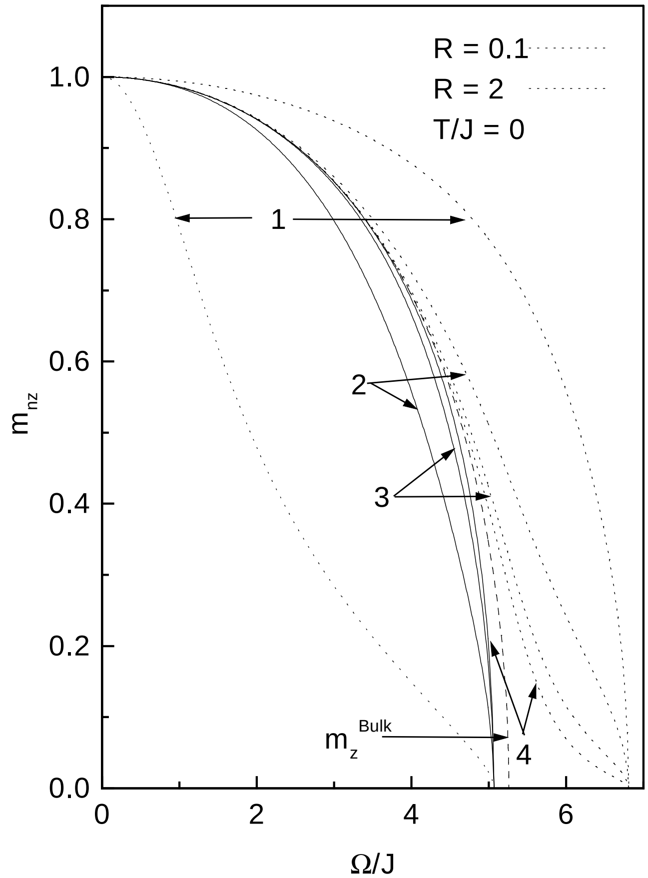
The longitudinal and transverse magnetizations and quadrupolar moments of the film are defined as the averages of the layer ones and are determined respectively from

$$\bar{m}_\alpha = \frac{1}{L}\sum_{n=1}^{L} m_{n\alpha} \tag{18}$$

and

$$\bar{q}_\alpha = \frac{1}{L}\sum_{n=1}^{L} q_{n\alpha}. \tag{19}$$

Because we are interested in the study of the longitudinal ordering in the transverse spin-1 Ising film, we limit our studies to the longitudinal order parameters (the longitudinal magnetizations and quadrupolar moments) and for simplicity, in the numerical calculation, we note that the symmetry of the film and thus the amount of calculation can be decreased greatly. After selecting some values of R, Ω/J and L, one can obtain the layer magnetizations and quadrupolar moments from equations (12)–(15) and their averages from equations (18) and (19).

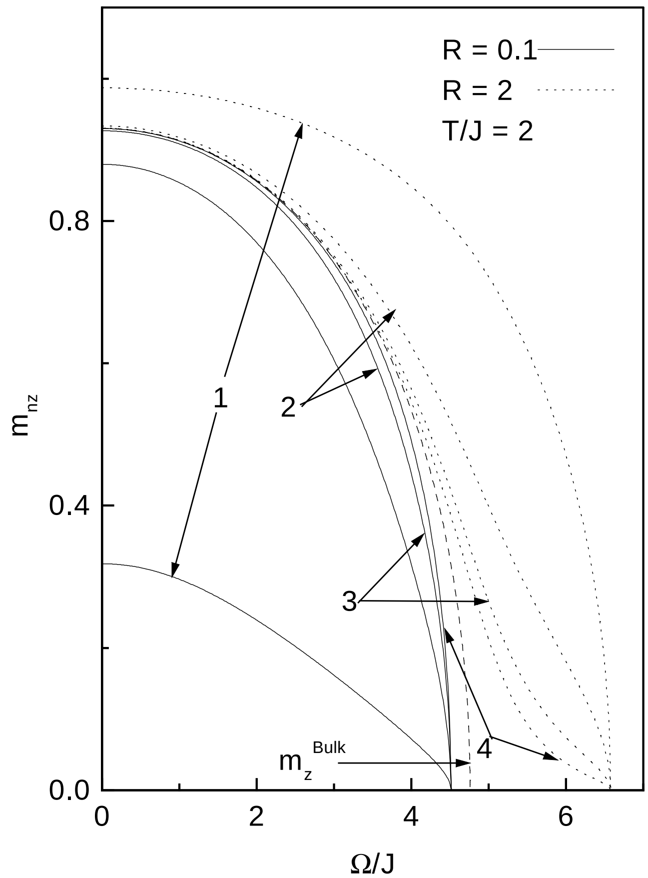


(a)

Figure 5. Layer longitudinal magnetizations as functions of the strength of the transverse field for a film with $L = 8$ layers. (a) $T/J = 0$, (b) $T/J = 2$. The solid and dotted curves correspond respectively to $R = 0.1$ and 2.

3. Order parameter curves

In this section, we are interested in the study of the layer longitudinal magnetizations and quadrupolar moments of a spin-1 Ising film in a transverse field. These quantities depend on the temperature, the ratio of the surface exchange interactions to the bulk ones $R = J_s/J$, the strength of the transverse field and the film thickness L. As far as we know, apart from the study by Zhong *et al* [7] of the critical behaviour of the transverse ferromagnet spin-1 Ising film, the order parameters of this model have not been studied in the past. In addition, the transverse field dependence of the order parameters of the transverse Ising systems has been considered only rarely [11, 26]. Therefore it is very interesting to investigate the temperature and transverse field dependences of the order parameters (the longitudinal magnetization and quadrupolar moment) of the transverse spin-1 Ising film. Because of the symmetry of the film, we give only $L/2$ layer order parameters as functions of temperature and transverse field in the corresponding figures. We will show only some typical results and we take the bulk exchange coupling J as the unit of energy in our calculations.



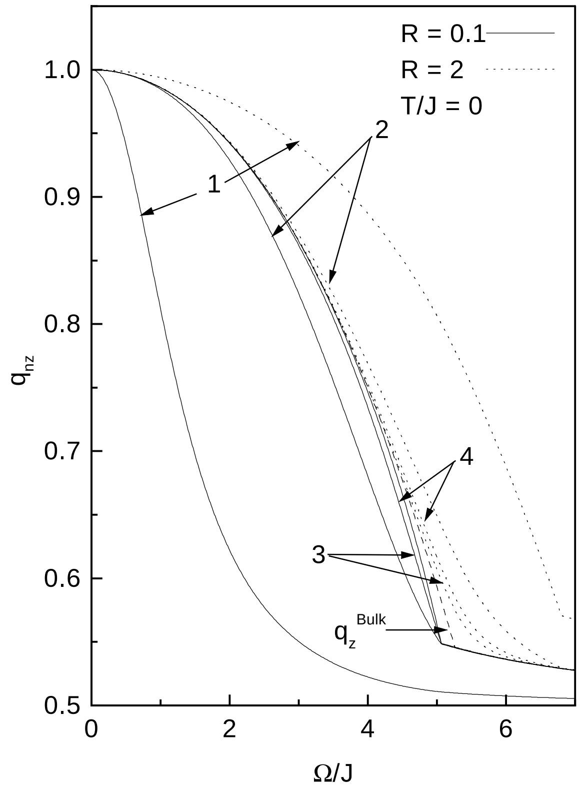
(b)

Figure 5. (Continued)

3.1. Temperature dependences of the order parameters

After selecting a value of the transverse field, a value of the ratio of the surface exchange interactions to the bulk ones and a value of the film thickness, one can obtain the layer longitudinal magnetizations and quadrupolar moments from equations (12)–(15) as functions of temperature, and then their averages from equations (18) and (19).

First we consider the variation of the film longitudinal magnetization $\bar{m}_z$ and the bulk magnetization m_z^{Bulk} of the system with the temperature. They are shown in figures 1(a) and 1(b) for two values of the ratio of the surface exchange interactions to the bulk ones $R = 0.1$ and 2 respectively, for various film thicknesses $L = 4$, 8 and 12 and for two values of the strength of the transverse field $\Omega/J = 0$ and 2. The dotted curves correspond to $\Omega/J = 0$ and the solid curves to $\Omega/J = 2$. It is clear from these figures that the magnetizations decrease with the increase of temperature and transverse field as expected. The bulk longitudinal magnetization vanishes at the bulk critical temperature T_c^B/J which depends on Ω/J only and the film longitudinal magnetization vanishes at the film critical temperature T_c/J which depends on R, Ω/J and L. We found that, depending on the value of the parameter R, the film critical temperature T_c/J may be lower or higher than that of



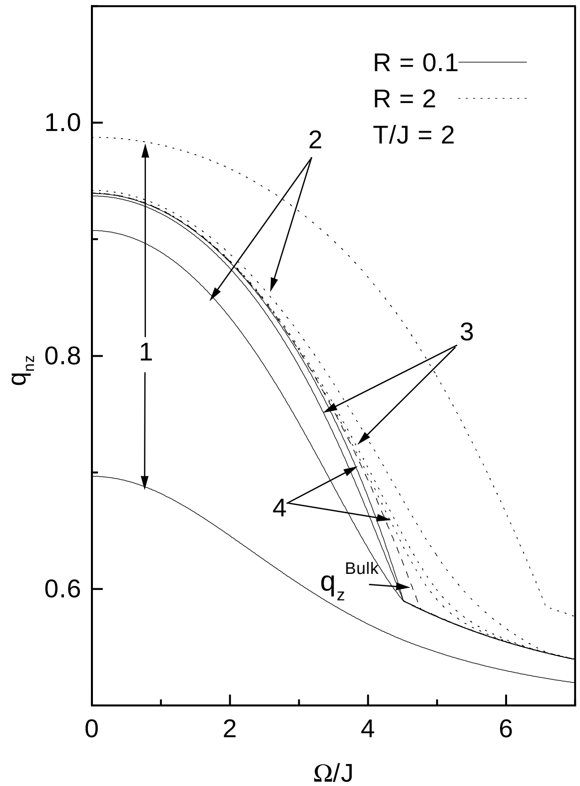
(a)

Figure 6. Layer longitudinal quadrupolar moments as functions of the strength of the transverse field for a film with $L = 8$ layers. (a) $T/J = 0$, (b) $T/J = 2$. The solid and dotted curves correspond respectively to $R = 0.1$ and 2.

the bulk T_c^B/J. This suggests that there exists a critical value R_c depending on Ω/J of the parameter R such that, for any given value of Ω/J, when $R < R_c$ the film critical temperature T_c/J is smaller than T_c^B/J and when $R > R_c$ the film critical temperature T_c/J is larger than T_c^B/J. From theses figures we see that for $\Omega/J = 0$ and $T/J = 0$, the bulk longitudinal magnetization and the longitudinal magnetizations of the film with $L = 4$, 8 and 12 are 1 (saturation magnetization $m_{z0} = 1$). For $R = 0.1$, the film longitudinal magnetization is smaller than the bulk longitudinal magnetization and, the thinner the film, the smaller the film longitudinal magnetization. For $R = 2$, the film longitudinal magnetization is larger than the bulk longitudinal magnetization and, the thinner the film, the higher the film longitudinal magnetization. To summarize the results, the film and bulk critical temperatures for two values of the transverse field, two values of R and for various film thicknesses are given in table 1.

In figures 2(a) and 2(b), we show the variation of the bulk longitudinal and the film longitudinal quadrupolar moments for the same values of the parameters as for figures 1(a) and 1(b) respectively. These quantities vary in the same way as the magnetizations, initially falling with the increase of the temperature at low temperatures and passing through a cusp at the transition temperature, before going to their asymptotic disordered-state value $\frac{2}{3}$. Here the case with $R = 0.1$ is slightly more sensitive to the transverse field strength.

The bulk and the layer longitudinal magnetizations are exhibited in figures 3(a) and 3(b)



(b)

Figure 6. (Continued)

Table 1. Summary of the results.

R	Ω/J	L	T_c/J	T_c^B/J
0.1	0	4	2.9392	3.5187
		8	3.3879	
		12	3.4627	
	2	4	2.7009	3.3229
		8	3.1840	
		12	3.2635	
2	0	4	4.6263	3.5187
		8	4.5664	
		12	4.5657	
	2	4	4.479	3.3229
		8	4.4180	
		12	4.4173	

for the case where the film thickness L is equal to 8, for $R = 0.1$ and $R = 2$, and for $\Omega/J = 0$ (figure 3(a)) and for $\Omega/J = 2$ (figure 3(b)). From these figures we see that the first layer or what is often called the surface layer has in the case where $R = 0.1$ ($R = 2$) the smallest (largest) longitudinal magnetization and all of the layer magnetizations are smaller (larger) than the bulk one. We observe also that the magnetization of the first layer is sensitive to the strength of the transverse field.

In figures 4(a) and 4(b), we show the variation of the bulk longitudinal and the film longitudinal quadrupolar moments for the same values for the parameters as for figures 3(a) and 3(b) respectively. These quantities vary in the same way as the magnetizations, initially falling with the increase of the temperature at low temperatures, and passing through a cusp at the transition temperature, before going to their asymptotic disordered-state value $\frac{2}{3}$. We observe also that the first-layer quadrupolar moment is more sensitive to the strength of the transverse field.

In order to clarify the effect of the strength of the transverse field on the layer order parameters, we have calculated many curves showing the variation of the order parameters with temperature for several values of the parameters R, Ω/J and L. All of the results are qualitatively the same and the main important result is that there should exist a critical value R_c of the parameter R depending on the value of Ω/J such that when $R > R_c$ the longitudinal magnetization and quadrupolar moment of the first layer are always greater than those of the internal layers which are also greater than those of the bulk system and when $R < R_c$ the opposite situation occurs. We remark also, from the above figures, that in all cases the layer longitudinal magnetizations vanish at the same transition temperature which is the film critical temperature which is smaller (larger) than the bulk critical temperature for $R = 0.1$ ($R = 2$). A critical value R_c at which the film and the bulk systems order simultaneously should exist. According to our results the critical value R_c of the parameter R depends on Ω/J and its values for $\Omega/J = 0$ and 2 are greater that 0.1 and less than 2. Here we do not evaluate the critical values of R_c.

3.2. The transverse field dependence of the order parameters

Solving equations (12)–(15) numerically, we can also obtain the layer longitudinal and transverse magnetizations and quadrupolar moments as functions of the transverse field. The variation of the order parameters with the strength of the transverse field have rarely been considered in the past [7, 26]. In this subsection, we assume the film to be composed of $L = 8$ atomic layers parallel to the surfaces as an example to help us to understand the transverse field dependences of the order parameters. The dependences of the bulk and the layer longitudinal magnetizations on the strength of the transverse field are exhibited for $R = 0.1$, 2 and for $T/J = 0$ in figure 5(a) and for $R = 0.1$, 2 and $T/J = 2$ in figure 5(b). The bulk critical transverse field Ω_c^B/J and the film critical transverse fields Ω_c/J at which the bulk and the film critical temperatures respectively go to zero are $\Omega_c^B/J = 5.2588$ and Ω_c/J equal to 5.0625 for $R = 0.1$ and 6.8039 for $R = 2$. These figures show that the magnetization of the surface layer is obviously different from those of the other layers. Figure 5(b) shows that, as the temperature is increased, the magnetization of the surface layer for $R = 0.1$ ($R = 2$) decreases (increases) rapidly. The variation of the bulk and the layer quadrupolar moments as functions of the strength of the transverse field are displayed for $R = 0.1$, 2 and for $T/J = 0$ in figure 6(a) and for $R = 0.1$, 2 and for $T/J = 2$ in figure 6(b). Comparing figure 6(a) with figure 6(b), we see that increasing temperature more obviously decreases the quadrupolar moments of the bulk system and of the film. Comparing the results obtained from figures 2(a) to 6(b), we see that for $R = 0.1$ in all cases the longitudinal magnetization of the surface layer is smaller than that of the second layer which is smaller than that of the third layer which is smaller than that of the fourth layer which is smaller than that of the bulk system and the same is true for the longitudinal quadrupolar moment. We have the opposite situation for $R = 2$. The results are in agreement with those of [26] and in disagreement with those of [11] obtained in the study of the layer longitudinal magnetizations of the transverse spin-$\frac{1}{2}$ Ising film.

4. Conclusions

In conclusion, using the effective-field theory with a probability distribution technique that accounts for the self-spin-correlation functions, we have studied the longitudinal magnetizations and quadrupolar moments of the spin-1 Ising film with a simple cubic structure in a transverse field. For a few selected sets of parameters we have calculated the averages and layer order parameters as functions of the temperature, the strength of the transverse field, the ratio R of the surface exchange interactions to the bulk ones and the film thickness and shown that there exists a critical value R_c depending on Ω/J such that when $R > R_c$ we have

$$m_{1z} > m_{2z} > \cdots > m_z^{\mathrm{Bulk}} \qquad q_{1z} > q_{2z} > \cdots > q_z^{\mathrm{Bulk}}$$

and when $R < R_c$ we have

$$m_{1z} < m_{2z} < \cdots < m_z^{\mathrm{Bulk}} \qquad q_{1z} < q_{2z} < \cdots < q_z^{\mathrm{Bulk}}.$$

The film possesses one well defined critical temperature which is smaller (larger) than the bulk one for $R < R_c$ ($R > R_c$) and, as for the bulk system, the film longitudinal magnetization vanishes and the quadrupolar moment passes through a cusp at the film critical temperature and they decrease with the increase of T/J or Ω/J. Other work that concerns the evaluation of the phase diagrams—such as the determination of R_c as a function of Ω/J for the spin-1 Ising film in a transverse field—is in progress and will be published.

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